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Complete Performance Space Modeling for Analog IC

Applying Pareto optimization for a successive approximation of all enclosing Performance Space surfaces

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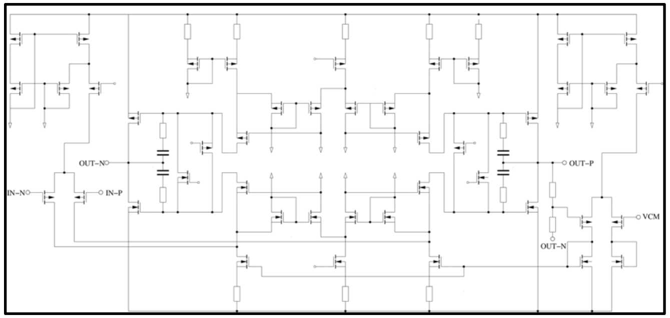
Outline

1. Introduction and Motivation

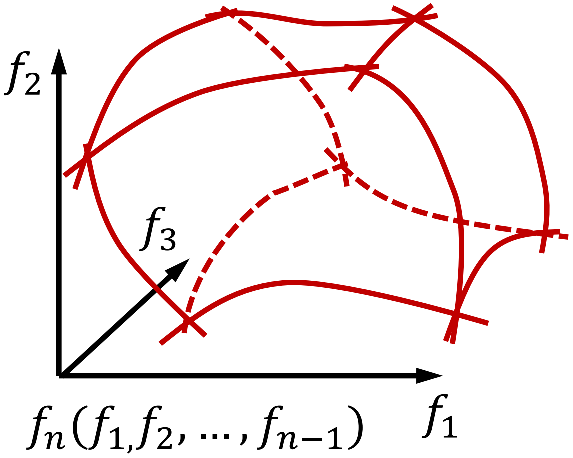
- Modeling dependencies of competing performances for an defined circuit topology
- Exploration of **all enclosing** boundaries of the feasible performance space



circuit netlist

Successive boundary approximation

by circuit sizing

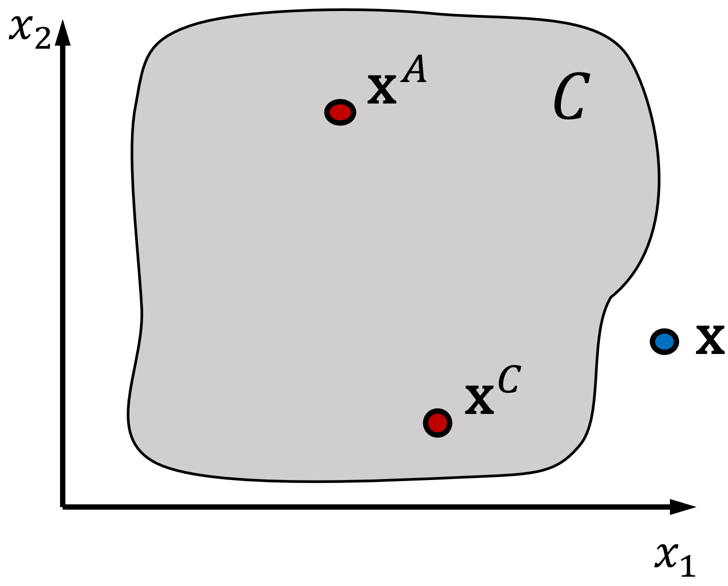


- Applications and Opportunities of Performance space models:
  - Circuit topology or technology comparison
  - Rating of structural circuit alternatives
  - High-level modeling of transistor level circuits

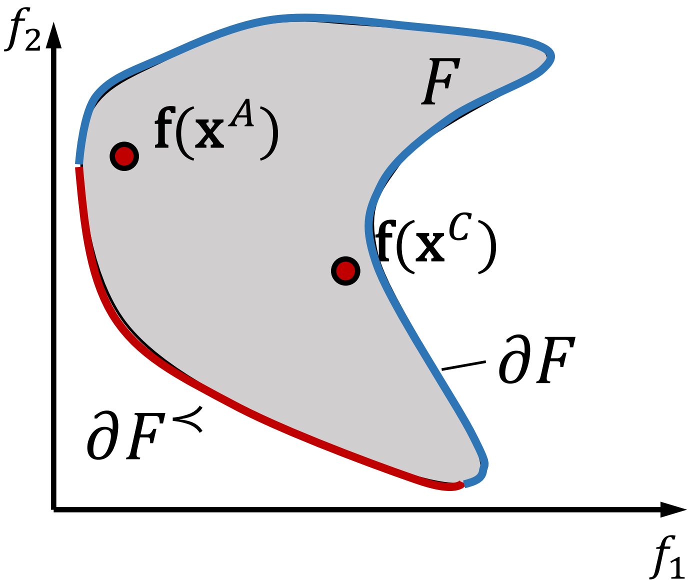
2. Complete Feasible P-Space

- Description of circuit performances in dependency to all other performances

Circuit Design Space



Circuit Performance Space



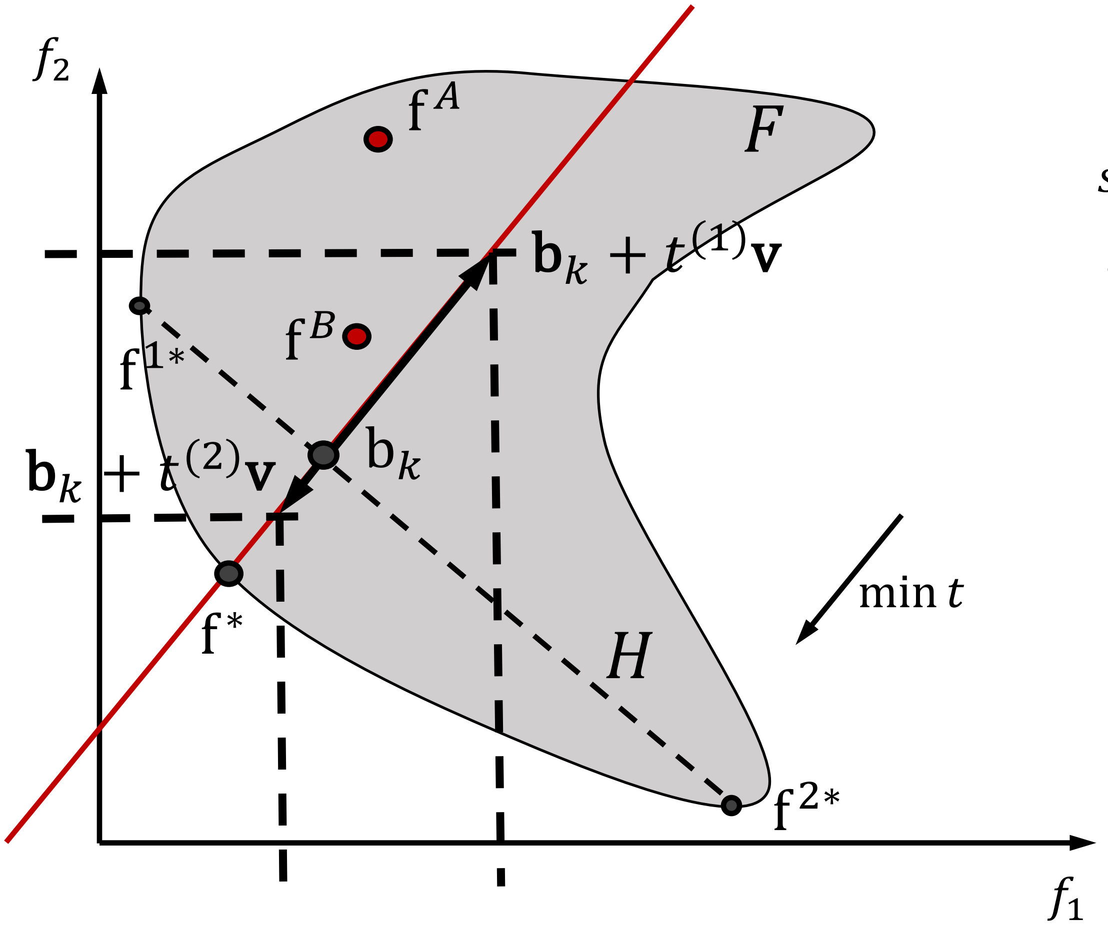
- $\mathbf{x} \in C$ (Constraint Space), if all electrical and geometrical constraints are fulfilled

🟥 Pareto Front

🟦 Dominated boundary

3. Normal Boundary Intersection

- Normal Boundary Intersection (NBI) [1,2,3,4,5]
- NBI using Goal Attainment, Feasible Wave Front SQP (TUM) [3]



- MOP:

$$\min_{x} F(x) \qquad s.t.\ \mathbf{x} \in \boldsymbol{C}$$

- SOP:

$$\min_{x,t} t$$

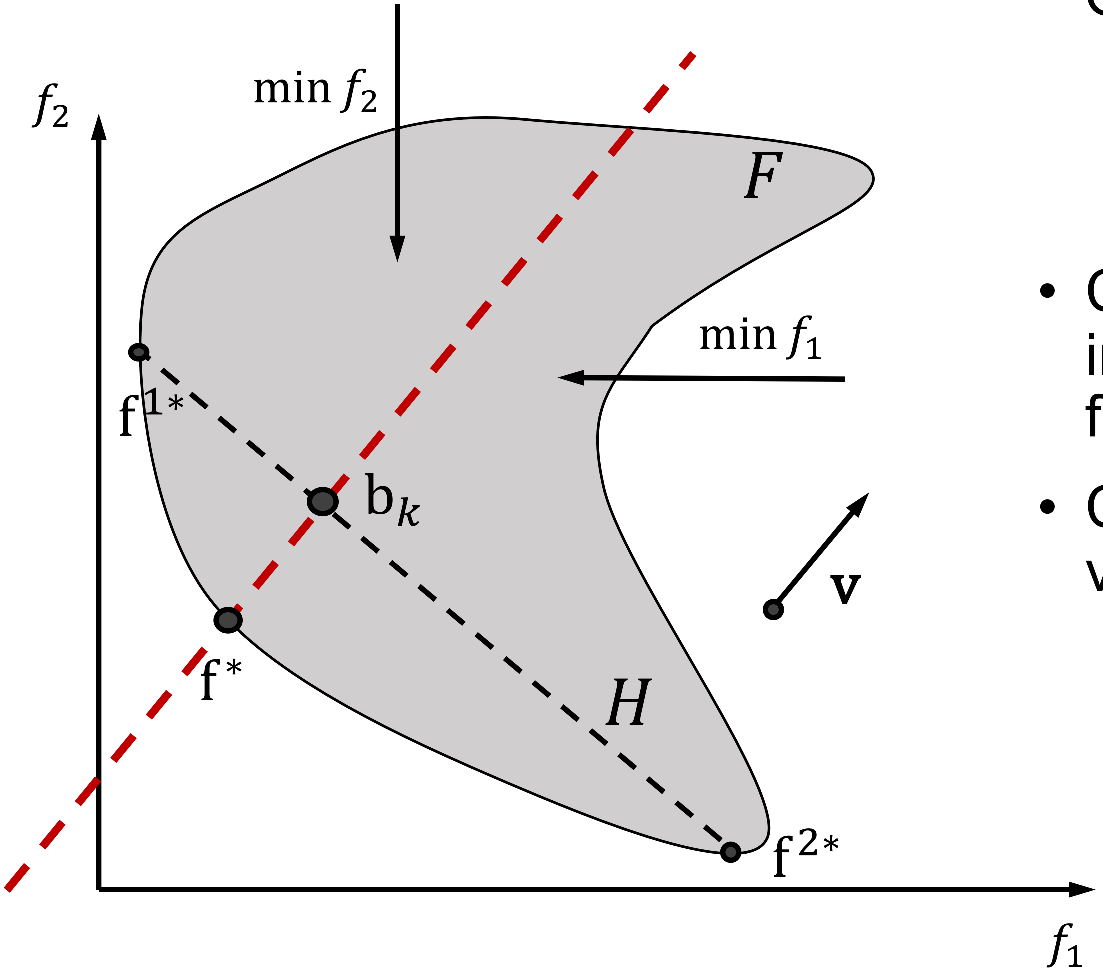
$$s.t. \quad \mathbf{f}(\mathbf{x}) \leq \mathbf{b}_k + t\mathbf{v}\,,\ \mathbf{x} \in \boldsymbol{C}\,, t\ \in \mathbb{R}$$

- Performance constraints along the search line, defined by the Base point $\mathbf{b}_k$ and a search direction $-\mathbf{v}$

$$b = \{H\mathbf{w}\colon \mathbf{w} \in \mathbb{R}^n, \sum_{i=1}^{n} w_i = 1, w_i \geq 0\}$$

- $t^{(1)}$: $\mathbf{f}^A$ *infeasible*
  $\mathbf{f}^B$ feasible
- $t^{(2)}$: $\mathbf{f}^A$ *infeasible*
  $\mathbf{f}^B$ *infeasible*

3. Normal Boundary Intersection



- Equivalent MinMax Optimization Problem [3]:

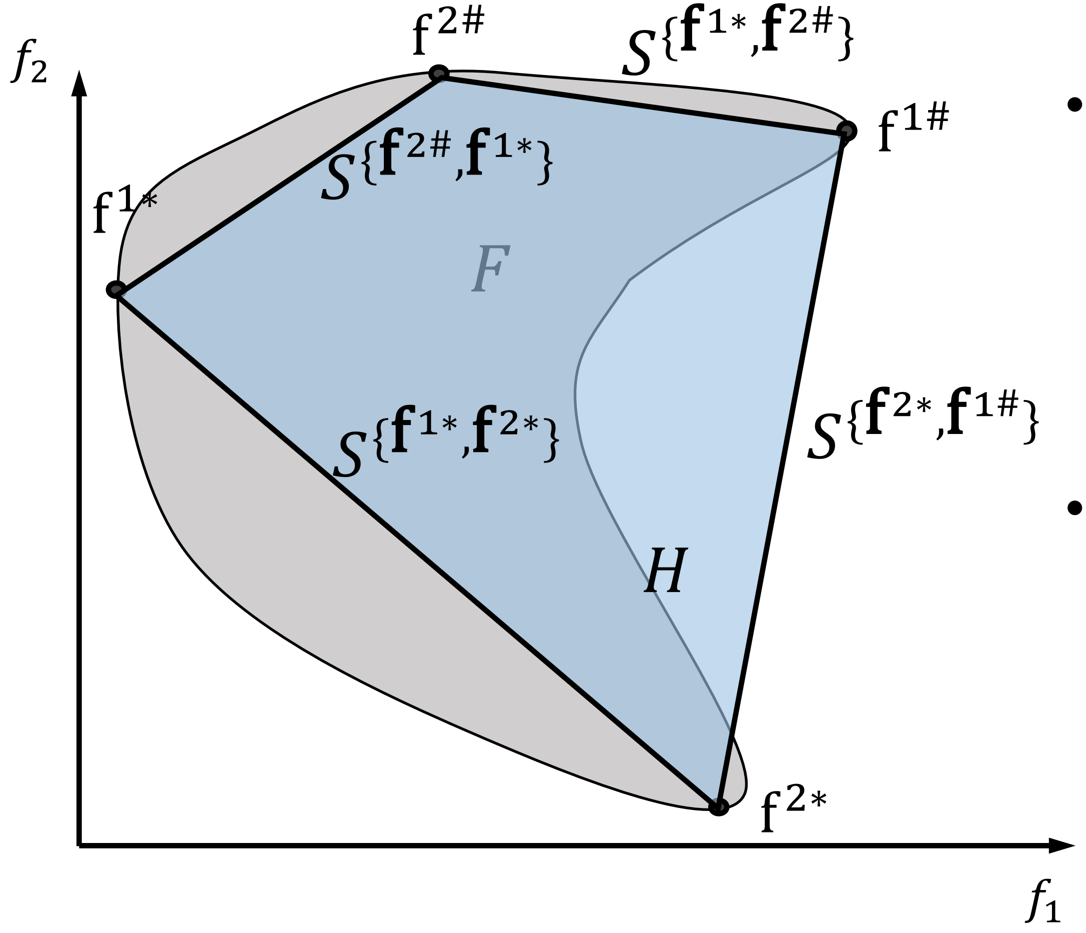
$$\min_x \max_i \left(\frac{f_i - b_i}{v_i}\right) \qquad s.t.\ \mathbf{x} \in \mathbf{C}$$

- Calculation of a well suited initial design parameter set for Goal Attainment
- Calculation of an initial value for t:

$$t = \max_i \left[\frac{f_i - b_i}{v_i}\right] \qquad s.t.\ \mathbf{x} \in \mathbf{C}$$

4. CFPS Algorithm

- **Step 1:** Calculation of all vertices, given by the individual minima and maxima for each performance parameter

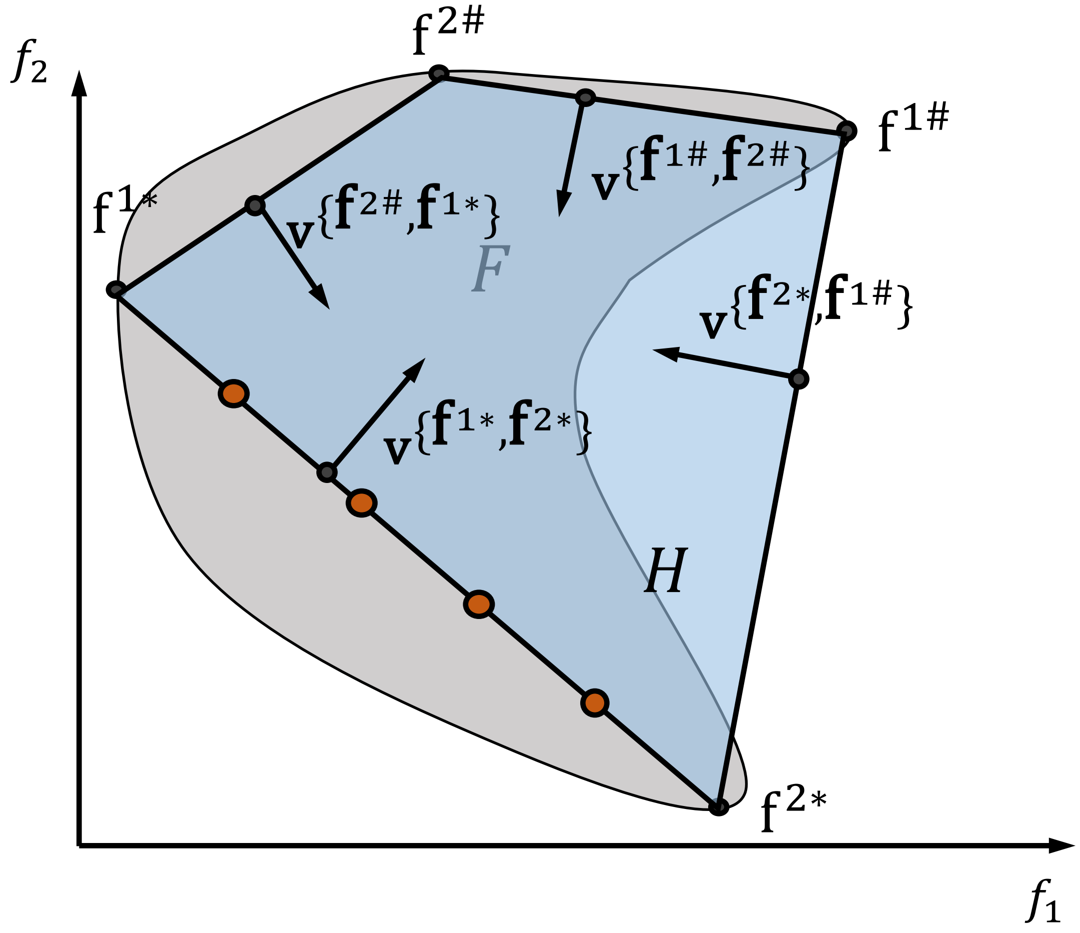


- **Step 2:** Determining the convex hull H of all maxima and minima (points, which are not in H are removed)

- **Step 3:** Defining Subproblems, by introducing Subspaces; all sets $S^{\{\dots\}}$ are subsets of H $(S \subseteq H)$

4. CFPS Algorithm

- **Step 4:** Determining the search direction for each Subproblem, given by the quasi normal vector $\mathbf{v}^{\{\cdots\}}$ of each convex subset S

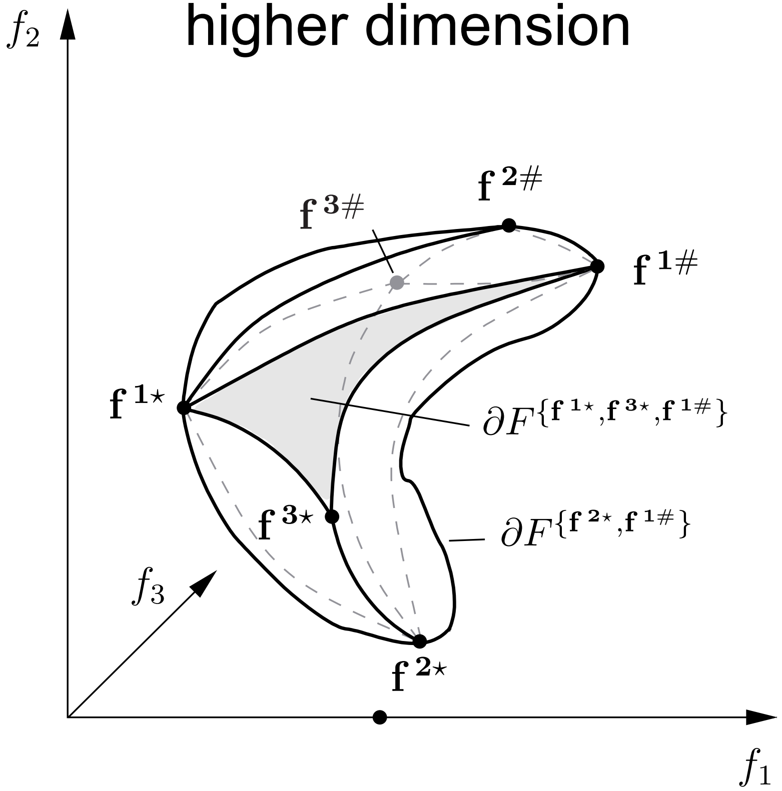


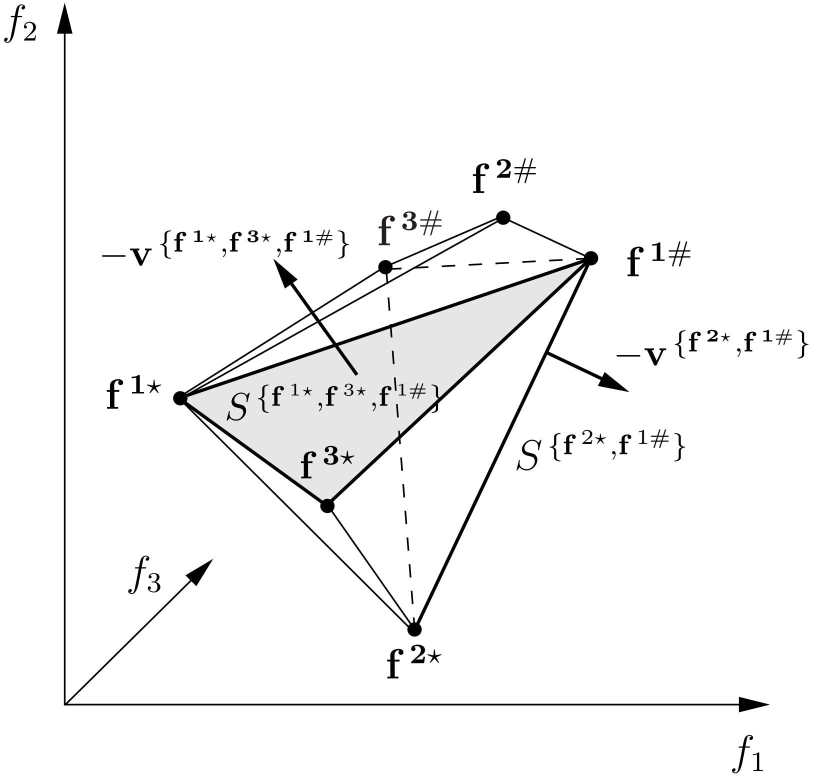
- Each vector $\mathbf{v}$ have to point towards the feasible space

- **Step 5:** Calculation of evenly distributed Base Points $\mathbf{b}_k$ between the associated vertices of the subproblem

4. CFPS Algorithm

- Subproblems for three performances and more:
  - In this case the vector $\mathbf{v}$ is given by the bisection vector between the $\mathbf{v}$'s of the adjoining neighbours with the next higher dimension





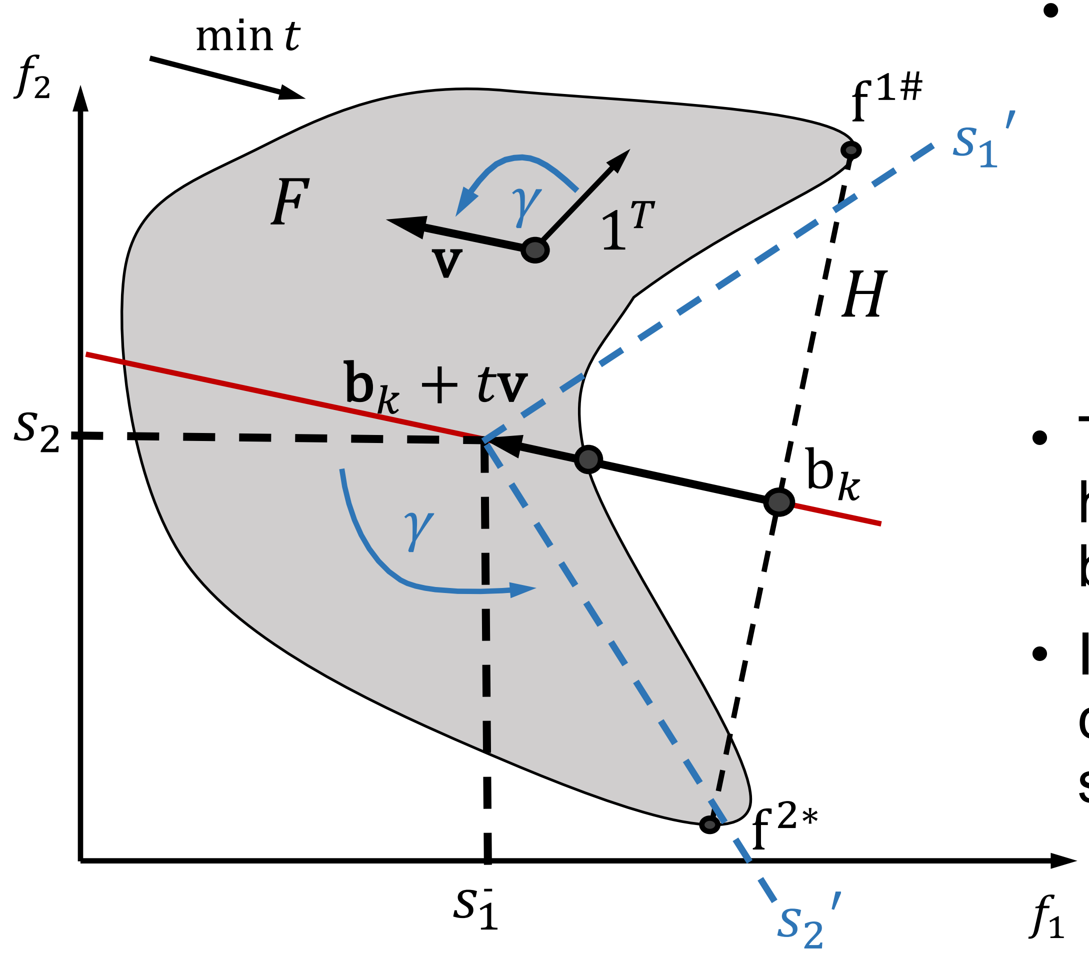
- **Step 5:** Deterministic evaluation of all Subproblems, starting with the problem of the smallest dimension.

4. CFPS Algorithm

- Calculation order of subproblems:
  - Vertices
  - Edges
    - perform all valid permutations of 2 vertices
  - Surface
    - perform all valid permutations of 3 edges
  - …
  - n-D hyperplane
    - perform all valid permutations of n hyperplanes of dimension $(n-1)$
  - Valid means → only permutations which lying on the associated superspace
- Number of Subproblems for n (worst case):
  - Max number of vertices: $2n$
  - Max number of n-D hyperplanes: $2n$

5. Modifications to NBI with GA

- Performance constraint rotation:
  - Enables the usage of NBI for the “rear side” of the CFPS



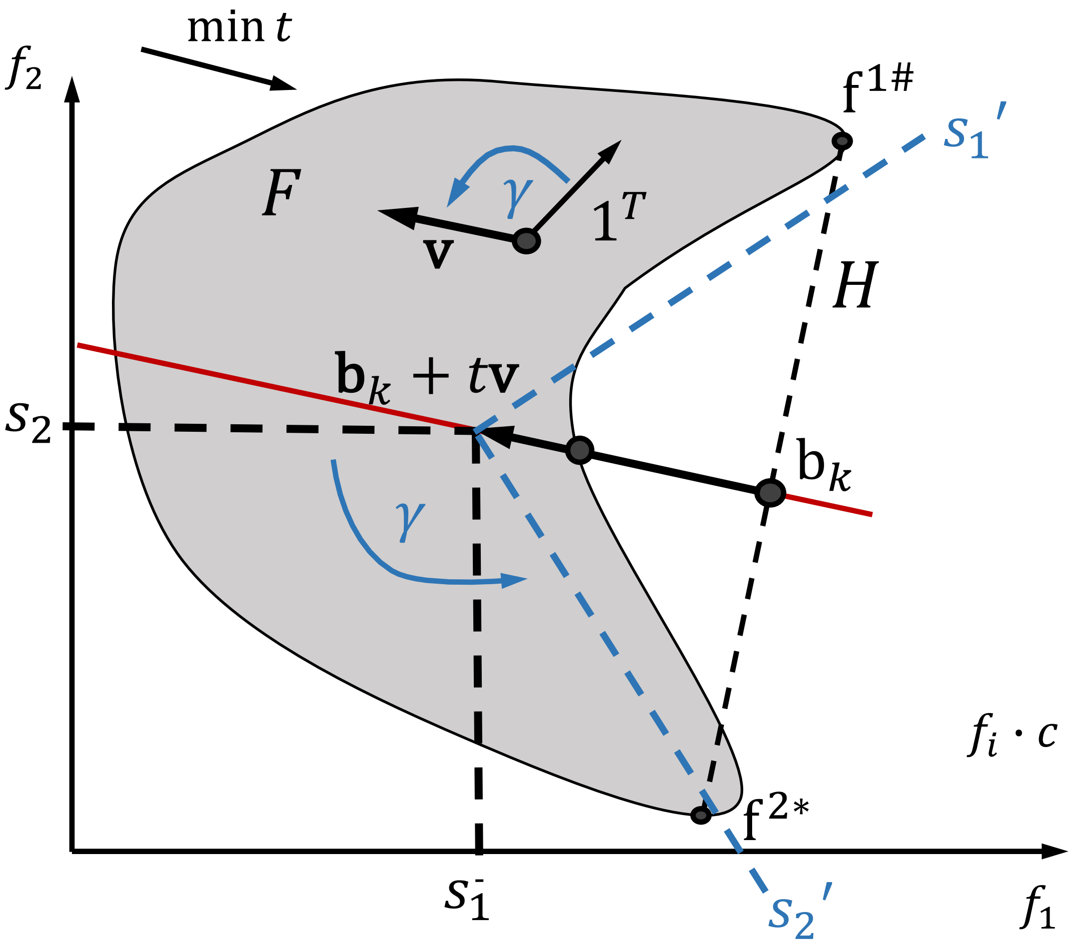
- Expression of every single performance constraint as linear equation (def. by their point of intersection):

$$s_i: \quad \mathbf{f} = \mathbf{b} + t\mathbf{v} + \lambda \mathbf{v}^{R_i}$$

- The new direction vector $\mathbf{v}^{R_i}$ holds the relative orientation between $\mathbf{v}$ and $\mathbf{1}^{\mathbf{T}}$
- In case $\gamma = 0$ the i-th component is ${v_i}^{R_i} = 0 \rightarrow$ standard NBI case

5. Modifications to NBI with GA

- Performance constraint rotation (2)



- Solving all linear equation systems s_i to f_i leads to:

$$f_i = b_i + tv_i - \frac{v_i{}^{R_i}(b_j + tv_j - f_j)}{v_j{}^{R_i}}$$

$$j \neq i$$

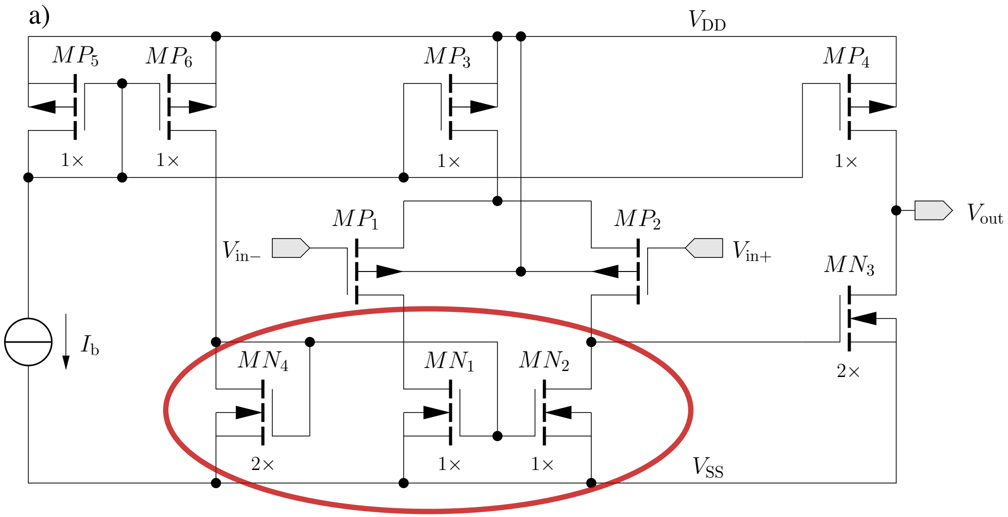
- Transformation to an inequation, by adding c, as orientation indicator between $\mathbf{v}$ and $\mathbf{1}^{\mathbf{T}}$ projected to the ij-plane.

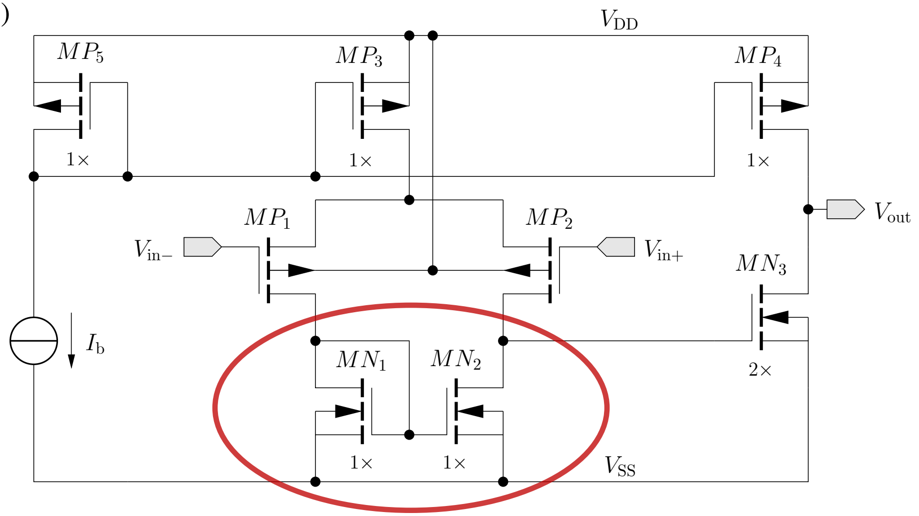
$$f_i \cdot c \leq \left(b_i + tv_i - \frac{v_i{}^{R_i}\left(b_j + tv_j - f_j\right)}{v_j{}^{R_i}} \right) c$$

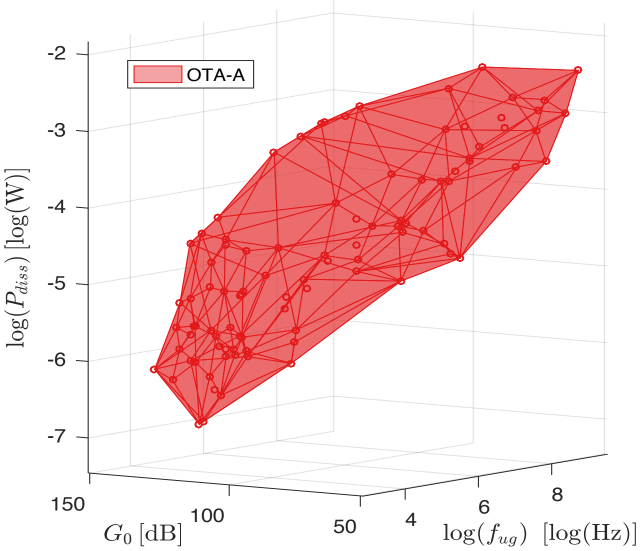
$$c = \; v_{ij} \circ v_{ij}{}^R$$

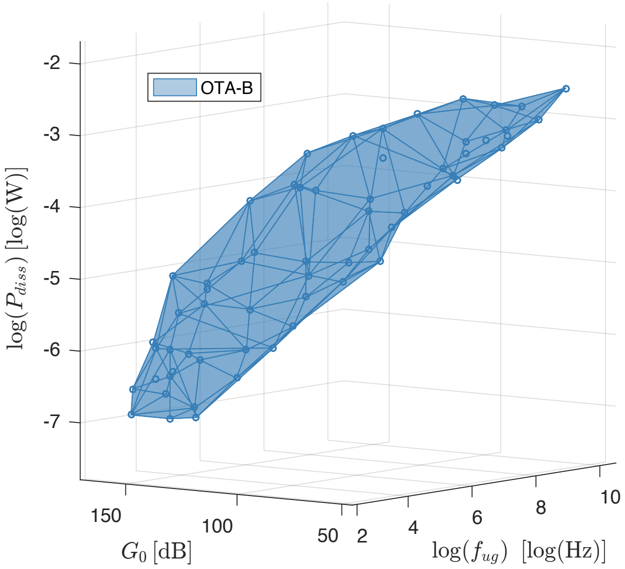
6. Example circuit

- 2-stage OTA in two versions:









7. Conclusion

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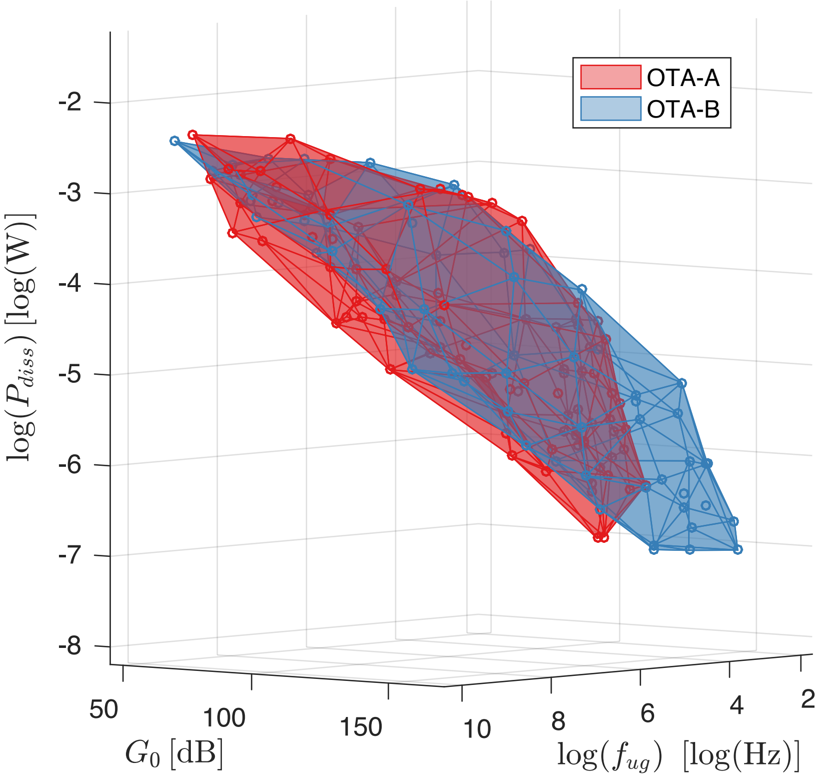
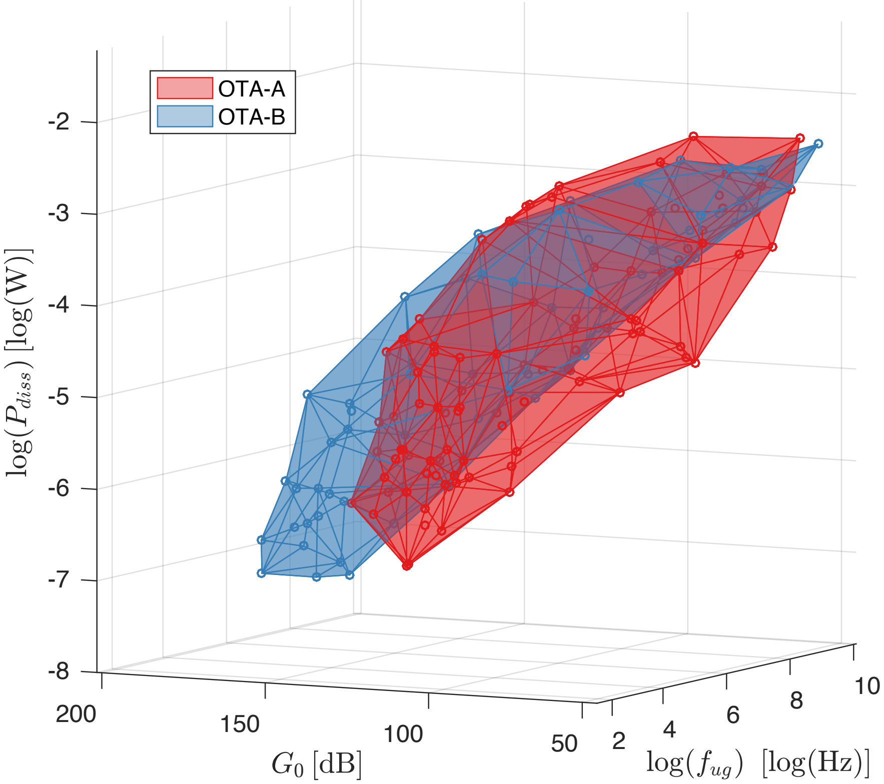
- Topology comparison:

Front view:



Rear view:



	OTA A	OTA B
Total number of boundary points	104	73
Number of vertices / edges / faces	5 / 9 / 6	4 / 6 / 4

7. Conclusion

- Performance space exploration by using NBI with some modifications
- Assumption of a closed boundary
- The effort depends on the surface area of the complete boundary → accuracy
- Tools: Matlab, WiCkeD core for circuit sizing

- Outlook:
  - Verification of the CFPS algorithm for more than three performances
  - Examination of correlation between computational effort and netlist size

Literature

[1] Das I, Dennis J (1998) Normal boundary intersection: a new method for generating Pareto surface in nonlinear multicriteria optimization problems. SIAM J Optim 8(3): 631–657

[2] Messac A, Ismail-Yahaya A, Mattson CA (2003) The normalized normal constraint method for generating the Pareto frontier. Struct Multidisc Optim 25(2): 86–98

[3] Mueller-Gritschneder D, Graeb H, Schlichtmann U (2009) A successive approach to compute the bounded Pareto front of practical multi-objective problems. SIAM J Optim 20(2):915–934

[4] Siddiqui S, Azarm S, Gabriel S (2011) On improving normal boundary intersection method for generation of Pareto frontier. Struct Multidisc Optim 43(6):839-852

[5] R. de S. Motta, S. M. B. Afonso, and P. R. M. Lyra A modified nbi and nc method for the solution of n-multiobjective optimization problems," Struct. and Multidisc Opt, 46, pp.239–259, Aug

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Thank you for your attention.

Research project:
EGEVATIS
(2017/05 – 2021/04)

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Performance space driven design of high accuracy low power evaluation circuits for thermoelectric two dimensional infrared sensors.