

# Complete Performance Space Modeling for Analog IC

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This publication shows an approach for a looped exploration of the complete feasible performance space of any given analog integrated circuit. The aim of the proposed scheme is to generate evenly spread points on the enclosing surfaces, based on spice-simulation results. To achieve this, well-known algorithms are selected to generate uniform Pareto optimal solutions. We are using the Normal Boundary Intersection method not only for optimal trade-off points, but also to explore the opposite and adjoining boundaries. As a result, we get a discrete evenly spread description of the complete feasible performance space. The method described is shown for two and three performances on an OTA circuit in CMOS technology. Advantages of such models could be a pre-design topology comparison.

## 1 Introduction

For designers of analog intergrated circuits, the first challenge is to select an useful topology to achieve the required specification. A description of the complete feasible performance space (CFPS) would prove very beneficial. The entirety of all design parameters describes the design space. Every design parameter combination produces a combination of performance parameters (PP) considering special constraints. The CFPS is described by all possible solutions, eq. (1). Most PPs compete with each other, so it leads to a multi-objective optimization problem (MOP). Since there does not exist one optimal solution to this problem, it can only be described by a trade-off curve, named Pareto optimal front. The idea proposed in this work, is to iterate over all individual surfaces generated between all meaningful combinations of the vertices. These individual subproblems (SP) will be performed with well-known Pareto point algorithms. Most used methodologies are the Normal Boundary Intersection (NBI), first mentioned in [1] and the Normalized Normal Constraint (NC) method, published in [3]. Since then several improvements were shown for the NBI in [4] and [2], especially on approximation of the bounded pareto front. In our Contribution, we are using the NBI method with the Goal Attainment [4], and adding some modifications, to fit also the "rear side" of the CFPS.

## 2 Methodology and Results

**Normalization of CFPS.** At first we calculate the individual minimum  $\mathbf{f}^*$  and maximum points  $\mathbf{f}^\#$  for each performance of interest. These points in eq. (2) are calculated with the weighted sum method, by setting one performance to one and the others to zero, first shown in [1]. If the optimization direction for any performance parameter is maximization, we have to transform it to a minimization problem, e.g. by multiplying with  $-1$ . Fig. 2(a) is showing a CFPS for two performances with pointing out the optimization direction. Dividing of each PP by its maximum spread leads to a normalized CFPS in fig. 2(b).

$$F = \{\mathbf{f} \in \mathbb{R}^n \mid \mathbf{f} = \mathbf{f}(\mathbf{x}) \wedge \mathbf{x} \in C\}, \mathbf{f} = [f_1, \dots, f_n]^T, C = \{\mathbf{x} \in \mathbb{R}^m \mid h(\mathbf{x}) = 0 \wedge g(\mathbf{x}) \leq 0 \wedge \mathbf{x}_l \leq \mathbf{x} \leq \mathbf{x}_u\} \quad (1)$$

$$\mathbf{f}^{i*} = [f_1(\mathbf{x}^{i*}), \dots, f_n(\mathbf{x}^{i*})]^T, \mathbf{f}^{i\#} = [f_1(\mathbf{x}^{i\#}), \dots, f_n(\mathbf{x}^{i\#})]^T, \quad \text{where, } i = 1, 2, \dots, n \quad (2)$$

**Solving Pareto Subproblems.** For each individual surface  $\partial F_{xx}$  we have to devise one Pareto-optimal problem. The transformation for the SP of surface  $\partial F_{00}$  is shown in fig. 2(c). All SPs are MOPs, which are solved by NBI using Goal Attainment [4]. An additional performance constrained is introduced, defined by a base point vector  $\mathbf{b}$  and a search direction vector  $\mathbf{v}$ . By minimizing the new design parameter  $t$ , a local trade-off point  $\mathbf{f}^*$  could be found for every base point. For the two-dimensional case, it have to be iterated over four SPs. The number of SPs is equal to the number of different combinations of all individual turning points. The worst case effort can be calculated by  $2^n$ .

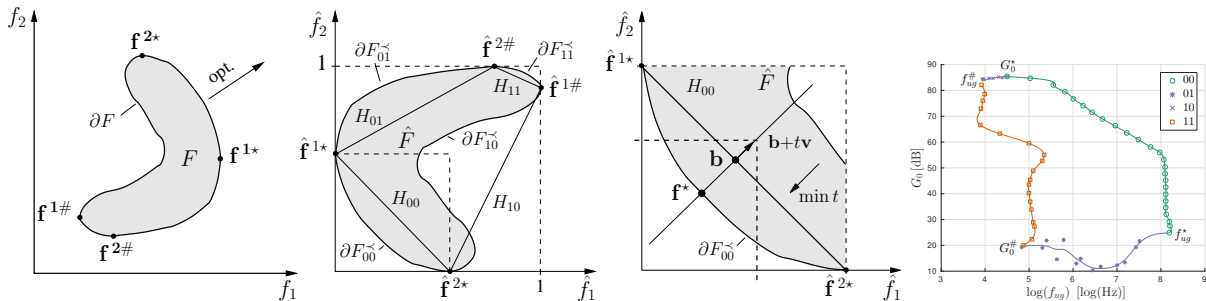


Figure 1: (a) CFPS with individual turning points; (b) Normalized CFPS; (c) Solving Subproblems with NBI and GA; (d) CMOS OTA example, Open loop gain vs. unity gain frequency

**Modification to NBI.** To achieve all adjoining boundaries of the feasible performance space, a rotation of the performance constraints is required. In this contribution, a rotation of each individual performance constraint, depending on the orientation of vector  $\mathbf{v}$ , is presented. So it is possible to calculate trade-off points on the mostly very concave "rear side" of the CFPS.

**Results.** The proposed scheme has been implemented for two performances on an OTA shown in fig. 2(d). Each plotted point in every SP is found by circuit sizing, using a feasible wave-front sequential quadratic programming algorithm.

## References

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